

Identifying Binomial Trials and Experiments

For an experiment to involve binomial trials, three conditions must be met:

- There must be two outcomes for each trial.
- The trials must be independent.
- The probability of a “success” must be the same for each trial.

Example: The experiment consisting of flipping a coin four times DOES represent binomial trials:

- There are two outcomes for each flip: heads and tails.
- Each flip is independent of the previous flips. That is, the result of one flip does NOT affect the result of a subsequent flip.
- The probability of a “success” remains the same for every flip. We can consider either heads or tails to be the success here; the probability of either occurring is 0.5.

For binomial trials to exist, the probability of each of “success” does NOT need to equal 0.5.

- Example: A coin could be bent in such a way that the probability of “heads” resulting is 0.6.
- Example: If an experiment were to be choosing a letter from the alphabet at random and a “success” was to draw the letter Q, the probability of choosing the Q would be $\frac{1}{26}$.

If an experiment appears to have more than two outcomes, it can sometimes be viewed in a different way.

- Example: Taking a multiple choice test with 4 choices per question and randomly choosing an answer. This scenario can also be viewed as having two options: getting each question correct or incorrect.



Exercise / Discussion: For each of the following experiments, describe whether the three conditions for binomial trials (two outcomes, independent trials, constant probability of a success) are met.

1. Guessing randomly for each question of a true/false quiz.
2. Determining the gender of the babies when triplets are born.
3. A bag contains 3 red and 3 blue marbles. Choose two marbles at random without replacing the marbles between draws.
4. Choosing two letters from the alphabet and determining whether each is a consonant or a vowel. Allow a letter to be chosen twice.

Reviewing Inequalities

- < less than
- ≤ less than or equal to
- > greater than
- ≥ greater than or equal to

In an algebra class, the inequality $x < 3$ refers to all real numbers which are less than 3, including decimals such as 2.98. This would be depicted on a number line as follows:



For the binomial problems in this unit we will not consider decimal values, so the whole number values less than 3 are {0, 1, 2}:



Exercise / Discussion: For each of the following inequalities, LIST all of the whole number values of x from 0 through 12 which satisfy the inequality.

5. $x < 7$

8. $x \geq 9$

6. $x > 10$

9. $2 < x < 7$

7. $x \leq 5$

10. $5 \leq x \leq 11$

Binomial Problems on the TI-84

There are two binomial functions on the graphing calculator, which can be found in the Distribution area of your calculator. Press the blue **2nd** key on your calculator, followed by the **vars** key to access the distr menu. You will be using choices A: and B: in your class, as shown at the right.



Depending on your calculator model, you will either see the screen at right

or you will only see "binompdf(" on your calculator screen. In this second case, use the following syntax to enter the values:



- Use **binompdf**(trials, p, x value) for $P(x = \#)$
 - This is ONLY used when you are finding the probability that x EQUALS some number.
- Use **binomcdf**(trials, p, x value) for $P(x \leq \#)$
 - This is ONLY used when you are finding the probability that x is LESS THAN OR EQUAL TO some number.
- For ANY other inequality, you will need to convert the problem into one containing a \leq symbol, as follows:

| Alternate Inequality | Use the following adjustment | Explanation |
|----------------------|---|--|
| $P(x < n)$ | Binomcdf (trials, p, $n - 1$) | Don't include n , but include $n - 1$ instead |
| $P(x > n)$ | $1 - \text{Binomcdf}$ (trials, p, n) | The "opposite" of $x > n$ is $x \leq n$ |
| $P(x \geq n)$ | $1 - \text{Binomcdf}$ (trials, p, $n - 1$) | The "opposite" of $x \geq n$ is $x \leq n - 1$ |
| $P(a \leq x \leq b)$ | Binomcdf (trials, p, b) - Binomcdf (trials, p, $a - 1$) | The first part yields all values $\leq b$, but we then subtract off the values below a |
| $P(a < x < b)$ | Binomcdf (trials, p, $b - 1$) - Binomcdf (trials, p, a) | The first part yields all values $< b$, but we then subtract off the values that are $\leq a$ |

Example: A woman gives birth to quadruplets. Assuming that the probability of a boy or a girl is equal, what is the probability that **exactly three** of the four quadruples are girls?

Preliminaries: Why is this a binomial problem?

- There are two possibilities for each child: boy or girl
- The gender of each child is independent of the others
- The probability of girl remains the same, 0.5, for each child.

Since we are finding the probability that there are EXACTLY three girls, we will use the binomPdf function.

We are seeking the find $P(x = 3)$. We will use binompdf (trials, p, x value)

- The number of trials is 4 because this problem involves quadruplets.
- The value of p is 0.5, the probability of a girl (which is our “success” in this problem).
- The x-value of interest in this problem is 3.



```
NORMAL FLOAT AUTO REAL RADIAN MP
binompdf
trials:4
p:.5
x value:3
```

Therefore, the probability is 0.25.



```
NORMAL FLOAT AUTO REAL RADIAN MP
binompdf(4,.5,3)
.....0.25
```

Example: A woman gives birth to quadruplets. Assuming that the probability of a boy or a girl is equal, what is the probability that **at most** three of the four quadruples are girls?

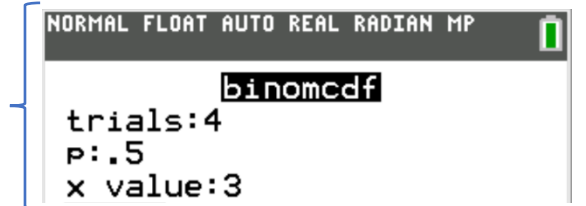
This is just like the previous example but contains the words “at most three” instead of “exactly.”

In this case, “at most three” refers to “three or less” which corresponds to the inequality “less than or equal to.”

Since we are finding the probability that there are AT MOST three girls, we will use the binomcdf function which is used specifically with \leq problems. (Note that “CDF” stands for “cumulative distribution function.”)


We are seeking the find $P(x \leq 3)$. We will use binomcdf (trials, p, x value)

- The number of trials is 4 because this problem involves quadruplets.
- The value of p is 0.5, the probability of a girl (which is our “success” in this problem).
- The x-value of interest in this problem is 3.



```
NORMAL FLOAT AUTO REAL RADIAN MP
binomcdf
trials:4
p:.5
x value:3
```

Therefore, the probability is 0.9375.



```
NORMAL FLOAT AUTO REAL RADIAN MP
binomcdf(4,.5,3)
.....0.9375
```



Exercise / Discussion: Which inequalities match with the phrases below?

At least fewer than at most between no more than no less than

Example: A salesperson shares her product at a swap meet to people who pass by her booth. From experience, there is a 22% chance that each contact will result in a sale. Find the probability that for a randomly selected day in which she makes 10 presentations, she makes...

- a. exactly 2 sales b. at most 3 sales c. fewer than 2 sales d. at least 3 sales e. between 2-8 sales

a. **exactly 2 sales:** This translates to $P(x = 2)$ since we see the word “exactly.” Therefore, we use the `binompdf` function without any adjustments. The number of trials is 10, and $p = 0.22$.

```
binompdf(10,0.22,2)
.....0.2984109099
```

Therefore, the probability of exactly 2 sales is 0.2984, or 29.84%.

b. **at most 3 sales:** This translates to $P(x \leq 3)$ because “at most” directly translates to the inequality “less than or equal to” which is precisely what is calculated when we use the `binomcdf` function. Again, the number of trials is 10, and $p = 0.22$.

```
binomcdf(10,0.22,3)
.....0.8413261068
```

Therefore, the probability of at most 3 sales is 0.8413, or 84.13%.

c. **fewer than 2 sales:** The phrase “fewer than 2” does not include the value of 2 sales. Instead, it only pertains to **0** or **1** sales. This translates to $P(x \leq 1)$ and NOT $P(x \leq 2)$ and we use the `binomcdf` function accordingly.

```
binomcdf(10,0.22,1)
.....0.3184693845
```

Therefore, the probability of fewer than 2 sales is 0.3185, or 31.85%.

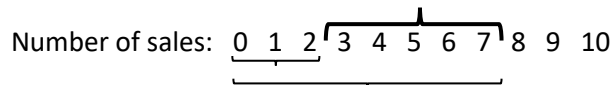
d. **at least 3 sales:** The phrase “at least 3” refers to making **3 or more** sales. This translates to $P(x \geq 3)$, but the calculator does not have a function designed to do inequalities of the “greater than or equal to” variety. Instead, we need to change the way we look at this problem. If 3 or more sales are made, this refers to making 3, 4, 5, 6, ... 10 sales. The opposite, or complement, of this would be to make 0, 1 or 2 sales; that is, the complement of “3 or more” is “2 or less.” We use the Complement Rule to calculate the probability: $P(\text{not } E) = 1 - P(E)$.

```
1-binomcdf(10,0.22,2)
.....0.3831197071
```

Therefore, we will calculate $1 - P(x \leq 2)$ to find the that the probability of at least sales is 0.3831, or 38.31%.

e. **between 2-8 sales:** The word “between” means that we do not include the endpoints of the interval. In this case, “between 2-8 sales” refers to making 3, 4, 5, 6 or 7 sales. Again, the graphing calculator does not have a built-in function for problems involving the word “between.” Instead, we must treat work this in two parts. We begin by calculating the probability of 7 or less sales, which will refer to making 0 though 7 sales. This, however, is not what we want: we only want the probability of 3-7 sales, not 0-7 sales. To remedy this, we subtract off the probability of 0 through 2 sales.

This is the interval of sales which we want for our calculations



We will first find $P(x \leq 7)$ and then subtract $P(x \leq 2)$ to find $P(3 \leq x \leq 7)$. We enter the following into the calculator: “`binomcdf(10, 0.22, 7) – binomcdf(10, 0.22, 2)`”

```
binomcdf(10,0.22,7)-binomcdf(10,0.22,2)
.....0.3829597857
```

Therefore, the probability of between 2-8 sales is 0.3830, or 38.3%.



Exercise / Discussion: Use the table in the middle of the page to help you determine what should be entered into the calculator to answer the following questions. Be prepared to explain your reasoning. Also calculate the requested probability.

11. A student guesses randomly on each question of a 20-question multiple choice test which offers 4 choices (A through E) for each question. Find the probability that the student correctly answers...

- a. exactly 5 questions
- b. less than 10 questions
- c. more than 8 questions
- d. at most 6 questions
- e. at least 6 questions
- f. between 2 and 6 questions
- g. between 4 and 9 questions, inclusive
- h. either 6 or 7 questions correct

| Alternate Inequality | Use the following adjustment | Explanation |
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| $P(a \leq x \leq b)$ | $\text{Binomcdf}(\text{trials}, p, b) - \text{Binomcdf}(\text{trials}, p, a - 1)$ | The first part yields all values $\leq b$, but we then subtract off the values below a |
| $P(a < x < b)$ | $\text{Binomcdf}(\text{trials}, p, b - 1) - \text{Binomcdf}(\text{trials}, p, a)$ | The first part yields all values $< b$, but we then subtract off the values that are $\leq a$ |

12. Example: A random sample of 40 items is taken off of an assembly line and inspected for defects. From historical data, it has been found the approximately 3% of the items are defective. Find the probability that the following number of items are defective...

- a. exactly two
- b. more than two
- c. between 1 and 5, inclusive