Math 226-DLA Limits Analytically

OBJECTIVE: Students will learn to find limits analytically by finding common denominators.

When to Use Common Denominators

If you are taking the limit of two fractions that are adding or subtracting or a complex fraction that contains two fractions that are adding or subtracting, try getting common denominators.

Common Denominator Examples:

EXAMPLE: $\lim_{x\to 1}\left(\dfrac{\frac{1}{2}-\frac{2}{x+3}}{x-1}\right)$

SINCE DIRECT SUBSTITUTION GIVES US A ZERO IN THE DENOMINATOR, WE MUST THINK OF OTHER TECHNIQUES WE CAN USE TO EVALUATE THE LIMIT. SINCE WE HAVE TWO FRACTIONS THAT ARE SUBTRACTING, LET'S TRY GETTING COMMON DENOMINATORS.

$$\lim_{x\to 1}\left(\dfrac{\frac{1}{2}-\frac{2}{x+3}}{x-1}\right)=\lim_{x\to 1}\left(\dfrac{\frac{1(x+3)}{2(x+3)}-\frac{2\cdot 2}{2(x+3)}}{x-1}\right)$$

GET COMMON DENOMINATORS.

$$=\lim_{x\to 1}\left(\dfrac{\frac{x+3-4}{2(x+3)}}{x-1}\right)$$

$$=\lim_{x\to 1}\left(\dfrac{\frac{x-1}{2(x+3)}}{\frac{x-1}{1}}\right)$$

$$=\lim_{x\to 1}\left(\frac{\cancel{x-1}}{2(x+3)}\cdot\frac{1}{\cancel{x-1}}\right)$$

$$=\lim_{x\to 1}\frac{1}{2(x+3)}$$

$$=\frac{1}{2(1+3)}$$

$$=\frac{1}{8}$$

EXAMPLE: $\lim_{x\to 0}\left(\frac{2}{x}-\frac{6}{x^2+3x}\right)$

SINCE DIRECT SUBSTITUTION GIVES US A ZERO IN THE DENOMINATOR OF BOTH FRACTIONS, WE MUST THINK OF OTHER TECHNIQUES WE CAN USE TO EVALUATE THE LIMIT. SINCE WE HAVE TWO FRACTIONS THAT ARE SUBTRACTING, LET'S TRY GETTING COMMON DENOMINATORS.

$$\lim_{x\to 0}\left(\frac{2}{x}-\frac{6}{x^2+3x}\right)=\lim_{x\to 0}\left(\frac{2}{x}-\frac{6}{x(x+3)}\right)$$

FACTOR THE DENOMINATOR.

$$=\lim_{x\to 0}\left(\frac{2(x+3)}{x(x+3)}-\frac{6}{x(x+3)}\right)$$

GET COMMON DENOMINATORS.

$$=\lim_{x\to 0}\left(\frac{2x+6-6}{x(x+3)}\right)$$

$$=\lim_{x\to 0}\frac{2\cancel{x}}{\cancel{x}(x+3)}$$

$$=\lim_{x\to 0}\frac{2}{x+3}$$

$$=\frac{2}{0+3}$$

$$=\frac{2}{3}$$

Try these on your own.

1. $\lim_{x\to 4}\frac{\frac{1}{x}-\frac{1}{4}}{x-4}$

2. $\lim_{x\to 0}\left(\frac{5}{x}-\frac{5}{x^2+x}\right)$

3. $\lim_{m\to 0}\frac{\frac{3}{m-1}+3}{m}$

4. $\lim_{h\to 0}\frac{\frac{1}{x+h}+\frac{1}{x}}{h}$

5. It will be helpful on your homework or an exam to quickly be able to recognize when you need to find common denominators. What are some characteristics of a function that would make you think to use this technique?