Math 226-DLA Limits Analytically

OBJECTIVE: Students will learn to find limits analytically by factoring and cancelling.

When to Factor and Cancel

If you are taking the limit of a quotient containing factorable polynomials or factorable trigonometric functions, try the factoring and canceling to remove the factor that is causing the quotient to be undefined.

Factor and Cancel Examples:

EXAMPLE: $\lim_{x \to 5} \frac{2x^2-7x-15}{x-5}$

SINCE DIRECT SUBSTITUTION GIVES US A ZERO IN THE DENOMINATOR, WE MUST THINK OF OTHER TECHNIQUES WE CAN USE TO EVALUATE THE LIMIT. SINCE WE HAVE QUOTIENT CONTAINING POLYNOMIALS, LET'S TRY FACTORING THE NUMERATOR.

THERE ARE MANY METHODS THAT WOULD WORK FOR FACTORING THIS. SINCE THERE IS AN $(x-5)$ IN THE DENOMINATOR, WE MIGHT USE THE "GUESS AND CHECK" METHOD WITH $(x-5)$ AS ONE OF OUR FACTORS. LET'S ALSO RELEARN THE AC-METHOD IN CASE WE COME ACROSS A FACTORING PROBLEM WHERE WE DON'T HAVE THIS TYPE OF A SITUATION TO GIVE US A HINT.

QUICK REMINDER OF HOW TO FACTOR

$2x^2-7x-15$	START BY MULTIPLYING A BY C, WHICH GETS US -30. WE ARE LOOKING FOR FACTORS OF -30 THAT ADD UP TO THE MIDDLE COEFFICIENT OF -7. THAT WOULD BE -10 AND 3. REWRITE THE MIDDLE TERM USING THOSE NUMBERS.
$2x^2-7x-15 = 2x^2-10x+3x-15$	NOW THAT WE HAVE FOUR TERMS, LET'S USE FACTORING BY GROUPING.
$= 2x(x-5)+3(x-5)$	FACTOR OUT THE COMMON FACTOR FROM THE FIRST TWO AND LAST TWO TERMS.
$= (x-5)(2x+3)$	FACTOR OUT THE GCF OF $(x-5)$.

$$\lim_{x \to 5} \frac{2x^2-7x-15}{x-5} = \lim_{x \to 5} \frac{(x-5)(2x+3)}{x-5}$$

$$= \lim_{x \to 5} \frac{\cancel{(x-5)}(2x+3)}{\cancel{x-5}}$$

$$= \lim_{x \to 5} 2x+3$$

$$= 2(5)+3$$

$$= 13$$

EXAMPLE: $\lim_{x \to 3} \frac{3-x}{x^2-9}$

SINCE DIRECT SUBSTITUTION GIVES US A ZERO IN THE DENOMINATOR, WE MUST THINK OF OTHER TECHNIQUES WE CAN USE TO EVALUATE THE LIMIT. SINCE WE HAVE QUOTIENT CONTAINING FACTORABLE POLYNOMIALS, LET'S TRY CANCELLING OUT COMMON FACTORS.

$\lim_{x \to 3} \frac{3-x}{x^2-9} = \lim_{x \to 3} \frac{-x+3}{(x+3)(x-3)}$

REARRANGE THE TERMS IN THE TOP SO THAT THE VARIABLE IS FIRST AND FACTOR THE DIFFERENCE OF SQUARES IN THE DENOMINATOR.

RECALL: $a^2 - b^2 = (a+b)(a-b)$

$= \lim_{x \to 3} \frac{-(x-3)}{(x+3)(x-3)}$

FACTOR OUT THE NEGATIVE FROM BOTH TERMS IN THE NUMERATOR.

$= \lim_{x \to 3} \frac{-\cancel{(x-3)}}{(x+3)\cancel{(x-3)}}$

COMMON ERROR: MANY STUDENTS CANCEL $(3-x)$ WITH $(x-3)$ IN THE FOLLOWING WAY $\frac{\cancel{3-x}}{(x+3)\cancel{(x-3)}}$. THEY ARE NOT THE SAME AND THEREFORE CANNOT BE CANCELLED AS SHOWN ABOVE.

$= \lim_{x \to 3} \frac{-1}{x+3}$

$= -\frac{1}{3+3}$

$= -\frac{1}{6}$

Try these on your own.

1. $\lim_{x \to -3} \frac{x^2-8x+12}{x^2+5x-14}$

2. $\lim_{x \to -4} \frac{x+4}{x^3+64}$

3. $\lim_{x \to -2} \frac{x^3+2x^2+4x+8}{x^2-4}$

4. $\lim_{x \to -3} \frac{x^2+3x}{2x^2+5x-3}$

5. Many types of factoring were used for the problems above—factoring out the GCF, the AC-Method, a difference of squares, a sum/difference of cubes and factoring by grouping. When you see the names of these factoring types mentioned, do you instantly know what they look like and know how to factor them? If any of the names are unfamiliar to you, look up and complete an example below.
6. It will be helpful on your homework or an exam to quickly be able to recognize when you need to factor and cancel. What are some characteristics of a function that would make you think to use this technique?