Math 226-DLA Limits Analytically

OBJECTIVE: Students will learn to find limits analytically by using special, well-known trigonometric limits.

When to Use Special Limits

We learned two special limits by looking at graphs and tables. Those were

$$\lim_{x\to 0}\frac{\sin(x)}{x}=1 \qquad \text{and} \qquad \lim_{x\to 0}\frac{1-\cos(x)}{x}=0.$$

Consider using these if the function you are finding the limit of contains a trigonometric function with a limit that approaches zero.

Special Limits Examples:

EXAMPLE: $\lim_{x\to 0}\frac{\sin 6x}{7x}$

SINCE DIRECT SUBSTITUTION GIVE US A ZERO IN THE DENOMINATOR, WE MUST THINK OF OTHER TECHNIQUES WE CAN USE THE EVALUATE THE LIMIT. THIS LOOKS SIMILAR TO ONE OF THE SPECIAL LIMITS THAT ARE TO BE MEMORIZED, SPECIFICALLY $\lim_{x\to 0}\frac{\sin(x)}{x}=1$. LET'S TRY USING THAT FACT TO EVALUATE THIS LIMIT.

$\lim_{x\to 0}\frac{\sin 6x}{7x}=\lim_{x\to 0}\left(\frac{1}{7}\cdot\frac{\sin 6x}{x}\right)$ **FACTOR OUT THE 1/7.**

$=\frac{1}{7}\cdot\lim_{x\to 0}\frac{\sin 6x}{x}$ **USE THE FOLLOWING PROPERTY OF LIMITS: $\lim_{x\to a}[c\cdot f(x)]=c\cdot\lim_{x\to a}f(x)$**

$=\frac{1}{7}\cdot\lim_{x\to 0}\left(\frac{6}{6}\cdot\frac{\sin 6x}{x}\right)$ **MULTIPLY BY 1 TO MAKE THE ARGUMENT OF SINE MATCH THE DENOMINATOR.**

$=\frac{1}{7}\cdot\lim_{x\to 0}\frac{6\sin 6x}{6x}$

$=\frac{1}{7}\cdot 6\cdot\lim_{x\to 0}\frac{\sin 6x}{6x}$ **FACTOR OUT THE 6 FROM THE NUMERATOR.**

$=\frac{6}{7}\,(1)$

$=\frac{6}{7}$

EXAMPLE: $\lim\limits_{x\to 0}\frac{\sin 3x+\tan x}{x}$

AGAIN, SINCE DIRECT SUBSTITUTION GIVE US A ZERO IN THE DENOMINATOR, WE MUST THINK OF OT1HER TECHNIQUES WE CAN USE THE EVALUATE THE LIMIT. IT LOOKS LIKE WE MIGHT BE ABLE TO USE SPECIAL LIMITS AGAIN.

$$\lim_{x\to 0}\frac{\sin 3x+\tan x}{x}=\lim_{x\to 0}\left(\frac{\sin 3x}{x}+\frac{\tan x}{x}\right)$$

$$=\lim_{x\to 0}\frac{\sin 3x}{x}+\lim_{x\to 0}\frac{\tan x}{x}$$

$$=\lim_{x\to 0}\frac{\sin 3x}{x}+\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{x}$$

$$=3\cdot\lim_{x\to 0}\frac{\sin 3x}{3x}+\lim_{x\to 0}\frac{\frac{\sin x}{\cos x}}{\frac{x}{1}}$$

$$=3(1)+\lim_{x\to 0}\left(\frac{\sin x}{\cos x}\cdot\frac{1}{x}\right)$$

$$=3+\lim_{x\to 0}\left(\frac{\sin x}{x\cos x}\right)$$

$$=3+\lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{1}{\cos x}\right)$$

$$=3+\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1}{\cos x}$$

$$=3+(1)\left(\frac{1}{\cos 0}\right)$$

$$=3+(1)(1)$$

$$=4$$

Try these on your own.

1. $\lim\limits_{x\to 0}\frac{\sin 5x}{4x}$
2. $\lim\limits_{x\to 0}\frac{1-\cos 2x}{x}$
3. $\lim\limits_{x\to 0}\frac{\tan 3x}{x}$
4. $\lim\limits_{x\to \pi/3}\frac{\sin x}{2x}$
5. It will be helpful on your homework or an exam to quickly be able to recognize when you need to use special limits. What are some characteristics of a limit that would make you think to use this technique?
6. Many students see problem 1 and 4 above as the same type. What makes them completely different techniques and which technique should be used on each?