

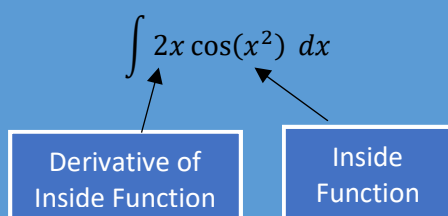
Math-226- DLA for U-Substitution

Name: _____

Objective: Learning to integrate using U-Substitution Method.

U-Substitution/ Substitution Method of Integration

“U- Sub” as it is called, is known as the reverse of the chain rule. In the chain rule, you must identify parts of the equation as the “Inside Function” and the “Outside Function.” In *u-Sub*, you must identify the inside function and the derivative of the outside function.



When to use Substitution Method

For an Integral to entail the use of U-Sub, The derivative of the inside function must be:

- Able to eliminate any other variables.
- Able to derive to a constant, so that the variable will be eliminated.
- In some Situations, either the inside or the derivative of the inside functions can be used as U

Common U-Sub Situations

$$\int 4x (x^2 + 5)^3 dx$$

$u = (x^2 + 5)$

$$\int \frac{x^3}{(2 + x^4)^2} dx$$

$u = (2 + x^4)$

$$\int x\sqrt{3x^2 + 4} dx$$

$u = (3x^2 + 5)$

$$\int x^3 e^{x^4} dx$$

$u = x^4$

Example 1:

$$\int 2x \cos(x^2) dx$$

Step 1: Identify u

$$u = x^2$$

Step 2: Take Derivative of u

$$du = 2x dx$$

Step 3: Apply formula to solve for dx

$$\frac{du}{2x} = \frac{2x dx}{2x}$$
$$dx = \frac{du}{2x}$$

Step 4: Replace x and dx

$$\int 2x \cos(u) \frac{du}{2x}$$

Step 5: Cancel out non- u variables.

$$\int \cancel{2x} \cos(u) \frac{du}{\cancel{2x}}$$

Step 6: Integrate in terms of u

$$\int \cos(u) du = \sin(u)$$

Step 7: Replace u and evaluate in terms of x

$$\sin(x^2) + C$$

U-Substitution Practice Problems

For exercises 1-5, integrate with given u.

1. $\int (x + 6)^{-2} dx$ $u = (x + 6)$
2. $\int t\sqrt{t^2 + 1} dt$ $u = (t^2 + 1)$
3. $\int \sin(4\theta - 7) d\theta$ $u = (4\theta - 7)$
4. $\int \sec^2(x) \tan(x) dx$ $u = \tan x$
5. $\int \frac{(\ln(x))^2}{x} dx$ $u = \ln(x)$

For exercises 6-10, integrate using u-substitution.

6. $\int \frac{5x^4 + 2x}{(x^5 + x^2)^3} dx$
7. $\int \theta \sin(\theta^2) d\theta$
8. $\int x^2(x^3 + 1)^{12} dx$
9. $\int_{-1}^2 \sqrt{5x + 6} dx$
10. $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cot \frac{x}{2} \csc \frac{x}{2} dx$

For exercises 11-14, complete the challenge problems (optional)

11. $\int x\sqrt{3x + 5} dx$
12. $\int 4xe^{x^2 + e^{x^2}} dx$
13. $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$
14. $\int_0^{\frac{\pi}{6}} \sec^2\left(2x - \frac{\pi}{6}\right) dx$